

A LOWER BOUND ON ADIABATIC HEATING OF COMPRESSED TURBULENCE FOR SIMULATION AND MODEL VALIDATION

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ABSTRACT

The energy in turbulent flow can be amplified by compression, when the compression occurs on a timescale shorter than the turbulent dissipation time. This mechanism may play a part in sustaining turbulence in various astrophysical systems, including molecular clouds. The amount of turbulent amplification depends on the net effect of the compressive forcing and turbulent dissipation. By giving an argument for a bound on this dissipation, we give a lower bound for the scaling of the turbulent velocity with compression ratio in compressed turbulence. That is, turbulence undergoing compression will be enhanced at least as much as the bound given here, subject to a set of caveats that will be outlined. Used as a validation check, this lower bound suggests that some simulations and models of compressing astrophysical turbulence are too dissipative. The technique used highlights the relationship between compressed turbulence and decaying turbulence.

1. INTRODUCTION

Turbulence undergoing mean compression, also called compressed turbulence, is of interest in a variety of disciplines. A number of studies, ranging from investigations of its essential behavior to detailed application studies, have been conducted with an eye towards internal combustion engines and aerodynamic flows. These include studies focusing on the zero-Mach-number limit (e.g. Morel & Mansour (1982); Wu et al. (1985); Coleman & Mansour (1991); Cambon et al. (1992); Guntsch & Friedrich (1996); Liu & Haworth (2010); Hamlington & Ihme (2014)), and those focusing on the finite-Mach-number limit (e.g. Blaisdell (1991); Speziale & Sarkar (1991); Durbin & Zeman (1992); Coleman & Mansour (1993); Cambon et al. (1993); Blaisdell et al. (1996); Grigoriev et al. (2016)).

Other contexts where compressed turbulence is of interest include: plasma physics and inertial fusion (Davidovits & Fisch 2016a,b; Thomas & Kares 2012; Weber et al. 2014; Kroupp et al. 2011, 2007a; Maron et al. 2013; Kroupp et al. 2007b), and astrophysics (Robertson & Goldreich 2012). In the astrophysics context, the turbulence undergoing compression is typically supersonic, and the present work focuses on compressed turbulence in this context.

Turbulence is ubiquitous in interstellar gas (Elmegreen & Scalo 2004), and the properties of supersonic turbulence have been related to important astrophysical questions, such as, the core mass and stellar initial mass functions (Padoan & Nordlund 2002; Ballesteros-Paredes et al. 2006; Hennebelle & Chabrier 2008), star formation efficiency (Elmegreen 2008), and the origin of Larson’s laws (Kritsuk et al. 2013a). As such, supersonic turbulence has been the subject of numerous investigations in the context of astrophysics (e.g. Mac Low et al. (1998); Mac Low (1999); Kritsuk et al. (2007); Federrath et al. (2008); Kritsuk et al. (2013b); Federrath (2013); Banerjee & Galtier (2014)). This astrophysical turbulence is often undergoing contraction or expansion under

the influence of gravity or pressure. Robertson & Goldreich (2012) pointed out that little work has been done on compressed turbulence in astrophysics, although intuition and some results from prior work on compressed turbulence in other contexts should be expected to carry over. Since contraction (or expansion) influences the behavior of the turbulence, and the turbulence plays a role in many problems related to interstellar gas dynamics, it is desirable to better understand how exactly contraction influences turbulent behavior.

At the most basic level, the first classifying parameter for turbulence undergoing compression is the ratio, $S = \tau_d/\tau_c$, of the turbulent dissipation time, τ_d , to the compression time, τ_c . When the compression is very slow, $S \ll 1$, and the compression has little effect. If the compression is very fast, $S \gg 1$, the turbulence is essentially “frozen” and its behavior can be treated with rapid distortion theory (RDT) (Savill 1987; Hunt & Carruthers 1990; Durbin & Reif 2010). For three dimensional rapid isotropic compressions, one finds that the root mean square (rms) turbulent velocity $v_{rms} \sim v_{rms,0}/\bar{L}$, where \bar{L} is the contraction factor along each axis, $\bar{L} = L/L_0$ (see e.g. Wu et al. (1985) for a zero-Mach RDT treatment, or Cambon et al. (1993) for a finite-Mach RDT treatment; a similar result is given by Peebles (1980) in section 90). In actuality, the turbulence will not be completely “frozen”, and there will be turbulent dissipation, the quantity of which depends (in part) on the rapidity of the contraction. This dissipation reduces the rms turbulent velocity scaling with compression below the $1/\bar{L}$ “adiabatic” result.

Here we present an argument for an upper bound on the amount of this turbulent dissipation, thereby providing a lower bound on the amount of adiabatic heating turbulence in a contracting gas can undergo. This argument rests on the following assumption. Consider as a base case the rate of decay for unforced Navier-Stokes (NS) turbulence with a constant viscosity. We assume that when the viscosity is a shrinking function of time, with the same initial value as the base case, the rate of decay is not larger than for the base case. If this

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physically-reasonable assumption holds, the bound follows directly. Then the bound can be used as a check on models and simulations of compressing turbulence, or as a model itself. We carry out an initial comparison with some previous work, which suggests that at least some approaches to simulating and modeling compressed high-Mach turbulence are too dissipative. They will give, for example, asymptotic scaling (in $\bar{L} \rightarrow 0$) of the turbulent velocity for a gravitational contraction that is below the minimum predicted by the bound. Since similar approaches are used in many astrophysical simulations, this apparent disagreement with the bound may have implications for other work as well.

The focus of the current work is to present the bound and an initial comparison against some previous work, thereby motivating future work to determine if the key assumption made in arriving at the bound holds. We note that even if some approaches to simulating and/or modeling compressed high-Mach turbulence are in fact too dissipative, a separate determination needs to be made as to whether this affects the results of interest. While physically reasonable, the assumption is not rigorous. In the surprising event the assumption is violated, so that the decay rate of NS turbulence is increased if the viscosity shrinks in time, there will likely still be implications for turbulence in astrophysical settings. Of course, the assumption (and bound) may hold in some situations and not in others, depending on the mechanism(s) by which the assumption is violated, if it is.

In the process of arriving at the bound, the sometimes forgotten relationship (Cambon et al. 1992) between turbulence forced by contraction and decaying turbulence is highlighted. Beyond its use in the argument for a lower bound, which is the focus of the present work, this connection may be helpful for understanding the influence of contraction on astrophysical processes, since it gives a means of translating quantities (e.g. correlation functions) between compressing and decaying cases. The relationship can also be useful for simplifying simulations of compressing turbulence (e.g. as used by Davidovits & Fisch (2016a)).

Although the bound presented here has a number of caveats associated with it, the approach used to arrive at it should be adaptable to create new bounds with different applicability. The bound is given in terms of \bar{L} , the Hubble parameter, $H = \dot{\bar{L}}/\bar{L}$ (with the overdot the time derivative), and the decay time constant t_0 and power α (in the spirit of Mac Low et al. (1998); Mac Low (1999)) for the rms velocity in decaying supersonic turbulence. The bound is

$$\frac{v_{rms}}{v_{rms,0}} \geq \frac{1}{\bar{L}} \left(1 + \frac{1}{t_0} \int_1^{\bar{L}} (\bar{L}')^{-3} \frac{d\bar{L}'}{H'} \right)^{-\alpha/2}. \quad (1)$$

As will be shown later, this form of the bound follows once a fit for the decay of v_{rms} in unforced NS turbulence (with a regular, constant viscosity) is chosen. If these fits are refined, the bound will be as well.

The paper is organized as follows. The model, essentially the NS equations in coordinates comoving with the contraction (or expansion), is described in Sec. 2. Section 3 shows the use of a time-dependent variable rescaling to change the NS equations forced by contraction

into NS equations for decaying turbulence, with extra time-dependent coefficients. An argument for the bound, Eq. (1), is given in Sec. 4, using the rescaled NS equations. Section 5 compares the bound to some previous results on compressing supersonic turbulence and discusses the caveats and implications of the bound and rescaling.

2. MODEL

The model is the Navier-Stokes equations written in contracting (or expanding) coordinates. These coordinates, \mathbf{x} , are defined in terms of the proper coordinates, \mathbf{r} , as

$$\mathbf{x} = \mathbf{r}/\bar{L}. \quad (2)$$

The proper velocity, \mathbf{u} , written in terms of the peculiar velocity \mathbf{v} and the contracting coordinates, is

$$\mathbf{u} = \dot{\bar{L}}\mathbf{x} + \mathbf{v}(\mathbf{x}, t) \quad (3)$$

Beginning with the NS equations for \mathbf{u} and the density ρ in the proper coordinates, and rewriting in terms of \mathbf{x} and \mathbf{v} , gives

$$\frac{\partial \rho}{\partial t} + \frac{1}{\bar{L}} \nabla \cdot (\rho \mathbf{v}) + 3 \frac{\dot{\bar{L}}}{\bar{L}} \rho = 0 \quad (4)$$

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\bar{L}} (\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{1}{\rho \bar{L}} \nabla p + \frac{1}{\bar{L}} \nabla \Phi + \ddot{\bar{L}} \mathbf{x} + \frac{\dot{\bar{L}}}{\bar{L}} \mathbf{v} = \frac{\mathbf{D}}{\bar{L}^2} \quad (5)$$

$$\frac{1}{\rho} (\mu \nabla^2 \mathbf{v} + (\mu + \lambda) \nabla (\nabla \cdot \mathbf{v})) = \mathbf{D}. \quad (6)$$

Here, p is the pressure, Φ is the gravitational potential, and \mathbf{D} is the usual dissipation term in the momentum equation, which is given in Eq. (6). It has been assumed that the dynamic and bulk viscosities, μ and λ respectively, are constants. A derivation of these equations, with the exception of the dissipation term, can be found in Peebles (1980), section 9. Essentially identical equations, based on contractions identical to those dictated by \mathbf{u} in Eq. (3), but without the gravitational potential, underly studies of compressing turbulence in other contexts (Blaisdell 1991; Cambon et al. 1993; Coleman & Mansour 1993).

Besides giving spatial derivatives time dependent coefficients (powers of \bar{L}), the effect of the contraction is to add forcing (or dissipation) to both the density and momentum equations. In the density equation, Eq. (4), the third term is a forcing term when $\dot{\bar{L}}$ is negative. This, in part, causes the mean density to increase as expected for the contraction.

In the momentum equation, Eq. (5), the first term to the left of the equals sign is similarly a forcing term when $\dot{\bar{L}}$ is negative. In fact, a similar term has been used as a way to add real space forcing for turbulence simulations (Lundgren 2003; Rosales & Meneveau 2005; Petersen & Livescu 2010). It is this term that, taken alone, will lead to the “adiabatic” increase of turbulent velocity $v_{rms} \sim 1/\bar{L}$.

The second term to the left of the equality in Eq. (5) is related to the acceleration of the contraction, $\ddot{\bar{L}}$. It depends on \mathbf{x} , and can cause the turbulence to be inhomogeneous (see Blaisdell (1991), section 2.4 for a thorough discussion). In the case where the contraction (the time dependence of \bar{L}) is determined by gravity, this acceleration term can be removed from the momentum equation

by the gravitational field of the mean density (see [Peebles \(1980\)](#)).

For the present work, we will treat this as the case, and we will also choose to ignore the gravitational effects associated with density fluctuations (as in [Robertson & Goldreich \(2012\)](#)). The pressure is taken to obey a polytropic law,

$$p = K\rho^\gamma, \quad (7)$$

with K and γ constants. Together, these choices give the model momentum equation,

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{L}(\mathbf{v} \cdot \nabla)\mathbf{v} + \frac{K}{\rho L}\nabla\rho^\gamma + \frac{\dot{L}}{L}\mathbf{v} = \frac{\mathbf{D}}{L^2}. \quad (8)$$

3. RESCALING

Substituting rescaled values of the density, velocity and time,

$$\rho = \bar{L}^\phi \hat{\rho}, \quad (9)$$

$$\mathbf{v} = \bar{L}^\delta \hat{\mathbf{v}}, \quad (10)$$

$$dt = \bar{L}^\tau d\hat{t}, \quad (11)$$

in the density and momentum equations, Eqs (4,8), gives

$$\frac{\partial \hat{\rho}}{\partial \hat{t}} = -\bar{L}^{\delta-\tau-1}\nabla \cdot (\hat{\rho}\hat{\mathbf{v}}) - \bar{L}^{-\tau}(3+\phi)H\hat{\rho} \quad (12)$$

$$\begin{aligned} \frac{\partial \hat{\mathbf{v}}}{\partial \hat{t}} = & -\bar{L}^{\delta-\tau-1}\hat{\mathbf{v}} \cdot \nabla \hat{\mathbf{v}} - \bar{L}^{-\delta-\tau+\phi(\gamma-1)-1}\frac{K}{\hat{\rho}}\nabla \hat{\rho}^\gamma \\ & - (1+\delta)\bar{L}^{-\tau}H\hat{\mathbf{v}} + \bar{L}^{-\phi-\tau-2}\hat{\mathbf{D}}. \end{aligned} \quad (13)$$

The Hubble parameter $H = \dot{L}/L$, and the dissipation $\hat{\mathbf{D}}$ is the same as in Eq. (6), but with $\hat{\rho}, \hat{\mathbf{v}}$ in place of ρ, \mathbf{v} .

By choosing $\phi = -3$ and $\delta = -1$, the forcing terms can be eliminated from the density and momentum equations. Then, choosing $\tau = -2$ removes the time dependent coefficient from the divergence term in the density equation, and also from the nonlinear term in the momentum equation. For these choices of ϕ, δ, τ , the incompressible case of this transformation has been discussed by [Cambon et al. \(1992\)](#). A different choice was made by [Davidovits & Fisch \(2016a,b\)](#), for the convenience of simulations. Various similarity transformations (e.g. [Nishitani & Ishii \(1985\)](#); [Nishitani \(1991\)](#); [Davis & Peebles \(1977\)](#)) are related.

We will also take the polytropic index $\gamma = 5/3$. Then, the rescaled NS equations become,

$$\frac{\partial \hat{\rho}}{\partial \hat{t}} = -\nabla \cdot (\hat{\rho}\hat{\mathbf{v}}) \quad (14)$$

$$\frac{\partial \hat{\mathbf{v}}}{\partial \hat{t}} = \hat{\mathbf{v}} \cdot \nabla \hat{\mathbf{v}} - \frac{K}{\hat{\rho}}\nabla \hat{\rho}^{5/3} + \bar{L}^3 \hat{\mathbf{D}}. \quad (15)$$

Up to the \bar{L}^3 scaling on the dissipation term, these are the usual, unforced, NS equations for a gas with polytropic index $\gamma = 5/3$.

4. BOUND

Turbulence governed by the rescaled equations, Eqs. (14,15), will decay, as it is unforced in these variables. The usual compressible NS equations are recovered by setting $\bar{L} = 1$. For contraction $\bar{L}(\hat{t}) \leq 1$ is a strictly decreasing function of time (the equality holds at

$\hat{t} = 0$). Since the viscous dissipation in Eq. (15) is multiplied by \bar{L}^3 , it has a smaller coefficient at all times after $\hat{t} = 0$ in the compressing case than in the $\bar{L} = 1$ usual case. Then it is reasonable to expect that the turbulent decay rate for the system Eqs. (14,15) will be slower than (or equal to) the decay for the usual compressible NS equations ($\bar{L} = 1$). This is the key assumption on which the bound rests.

If this assumption is true, then the rms turbulent velocity for the system, Eqs. (14,15), will be at least as great as that given by the usual power law decay for the system when $\bar{L} = 1$,

$$\hat{v}_{rms} \geq \hat{v}_{rms,0} (1 + \hat{t}/t_0)^{-\alpha/2}. \quad (16)$$

Here, α and t_0 are to be determined for turbulent decay in the non-compressing ($\bar{L} = 1$) case. Then, arriving at the bound, Eq. (1) requires using Eqs. (10,11) to transform Eq. (16) into the unscaled (non-hat) variables. One could instead write a comparable bound for the turbulent kinetic energy (TKE), $\langle \rho v^2/2 \rangle$, under compression. We use the turbulent velocity in keeping with previous work ([Robertson & Goldreich 2012](#)). If a decay law of a different form than that given by Eq. (16) is more appropriate, there will still be an equivalent bound, derived once again by transforming the decay law back to the unscaled variables.

Although we are not aware of work determining t_0 and α for the rms velocity decay of supersonic turbulence with $\gamma = 5/3$, we can estimate these values from closely related work. The bound will then be only a guide. [Mac Low \(1999\)](#) finds that for supersonic (initially mach 5) isothermal decaying turbulence, t_0 is the initial turnover time for the turbulence (at the driving scale). [Mac Low et al. \(1998\)](#) find that the TKE in supersonic (mach 5) turbulence with $\gamma = 7/5$ decays with power $\alpha \sim 1.2$. In the isothermal case ($\gamma = 1$), they found $\alpha \sim 1$, suggesting some slight dependence on γ , at least within this modest range. [Smith et al. \(2000\)](#) find for the decay of hypersonic (mach 50) isothermal turbulence a decay power $\alpha \sim 1.5$, after an initial transient period. While these results suggest a single value of α will not suffice for all situations, we may expect that for $\gamma = 5/3$, α is roughly in the range $1 \sim 1.5$, depending on the initial mach number.

Note that these decay rates are for the TKE, not the rms velocity. Using them for the decay of the rms velocity discounts density-velocity correlations. [Mac Low \(1999\)](#) finds these correlations make for a 10-15 percent difference between the TKE calculated from the rms velocity, $mv_{rms}^2/2$, and TKE calculated directly, $\langle \hat{\rho} \hat{v}^2/2 \rangle$. Again, this result is for mach 5 turbulence, and may change with mach number.

5. DISCUSSION

The bound, Eq. (1), can be used as a validation tool. For example, let us compare it with the compressing turbulence model and matching simulations of [Robertson & Goldreich \(2012\)](#). That model is

$$\frac{dv_{rms}}{dL} = - \left[1 + \eta \frac{v_{rms}}{HLL_0} \right] \frac{v_{rms}}{L}. \quad (17)$$

This model for v_{rms} includes two components: the forcing due to the contraction (the first term left of the

equals sign in Eq. (5)), and the dissipation of v_{rms} calculated from the equilibrium dissipation rate for forcing at a given scale, as found by Mac Low (1999). The forcing scale is taken to decrease in time as determined by the contraction, \bar{L} . Robertson & Goldreich (2012) find that $\eta = 1.2$ creates a good match between the model and their simulation results, which were carried out for isothermal turbulence. The model is nominally independent of γ , although to the extent the turbulent dissipation rate depends on γ , one may expect that η could change.

Assuming equality in the bound, Eq. (1) and differentiating with respect to \bar{L} , one can write an expression for v_{rms} that is comparable to Eq. (17),

$$\frac{dv_{rms}}{d\bar{L}} = - \left[1 + \frac{\alpha}{2t_0} \frac{\bar{L}^{2/\alpha-2}}{H} \left(\frac{v_{rms}}{v_{rms,0}} \right)^{2/\alpha} \right] \frac{v_{rms}}{\bar{L}}. \quad (18)$$

For $\alpha = 2$, $\eta = 1$, and taking the initial turnover time as $t_0 = L_0/v_{rms,0}$, the two expressions, Eq. (17) and Eq. (18), are equal. The bound and the model of Robertson & Goldreich (2012) are calculated for different values of γ , complicating a direct comparison. However, using best estimates for the values of α and η in a comparable case, it seems the model, Eq. (17), and matching simulations, are too dissipative, predicting a v_{rms} below the lower bound. To compare, consider the $\gamma = 5/3$ case, for which the bound is calculated. From the discussion of values of α in Sec. 4, it seems reasonable to expect that, for initially mach 6 turbulence, with an adiabatic index $\gamma = 5/3$, the decay constant α is much closer to 1 than to 2. Since the decay rate is relatively insensitive to changes in γ for turbulence of similar initial mach number (mach 5, Mac Low et al. (1998)), we expect the value of $\eta = 1.2$ found by Robertson & Goldreich (2012) will not change drastically. Then, if $\eta \sim 1.2$ and $\alpha \sim 1 - 1.5$, v_{rms} predicted by the model, Eq. (17), will be below the lower bound. This indicates the simulations (to which the model is fit), are too dissipative.

This difference will impact, for example, the asymptotic scaling of v_{rms} with \bar{L} . As the methods used to arrive at both the model, Eq. (17), and the bound, Eq. (1), seem reasonable, reconciling this difference requires more detailed consideration.

Assuming equality in the bound, Eq. (1), is equivalent to asserting that the time dependent pre-factor (\bar{L}^3) of the viscous dissipation, $\hat{\mathbf{D}}$, in Eq. (15) does not decrease the dissipation rate of the turbulence, despite the fact that the coefficient decreases in time. That is, that the dissipation rate (and therefore energy behavior) of the turbulence is independent of time dependence in the viscous coefficient. For various subsonic compressing turbulence studies, this has not been found to be the case (Coleman & Mansour 1991; Cambon et al. 1992; Davydovits & Fisch 2016b). Since dissipation in decaying supersonic (isothermal) turbulence is primarily in shocks (Smith et al. 2000), it is conceivable the situation changes between subsonic and supersonic turbulence. Further, in the previously studied subsonic cases, the viscous coefficient is generally increasing in time, rather than decreasing as in the present situation.

If the shrinking-in-time dissipation coefficient did have no impact on the dissipation rate, then Eq. (1), with

equality assumed, would be a model for v_{rms} , rather than a bound. Furthermore, in this case, for a given initial condition, a single simulation of Eqs. (14,15) would be sufficient for *all* compression histories $\bar{L}(t)$ (or Hubble parameters, $H(t)$, alternatively). This is because the Eqs. (14,15) would no longer have any dependence on \bar{L} .

If the shrinking dissipation coefficient counter-intuitively led to *more* dissipation than in the case where the coefficient is constant, the lower bound would be invalid. If this effect were consistent, it would instead represent an upper bound (with \geq in Eq. (1) switching to \leq).

The bound depends to some degree on the choice of physical model for the dissipation process. If the relevant dissipation process is not captured by the NS viscous dissipation, Eq. (6), the bound may change. This is because the time-dependent coefficient of the dissipation in the rescaled momentum equation, Eq. (15), results from transformation and rescaling of the dissipation. For a different dissipation form, one could imagine that the coefficient after rescaling is different from the \bar{L}^3 coefficient found here. This could alter the bound, particularly if the coefficient were no longer shrinking in time. Additionally, the form of the NS dissipation, \mathbf{D} , doesn't change under transformation to the moving frame and rescaled variables, allowing the analogy between the compressing case and the uncompressing case.

This will not necessarily be true for all imaginable dissipation forms. For example, if the physically correct dissipation for the fluid equations took the form of the artificial viscosity commonly used for shock-capturing (see e.g. VonNeumann & Richtmyer (1950); Stone & Norman (1992)), the bound would need to be reconsidered. Note that, for numerical simulations in the moving frame (solving Eqs. (4,5)), the form of the dissipation may need to be considered explicitly, as done here, so that its transformation can be accounted for.

We now turn to the dependence of the bound on the adiabatic index. When $\gamma \neq 5/3$, the scaled momentum equation, Eq. (15), will pick up additional time dependence, as a coefficient for the pressure gradient term. This worsens the analogy between the scaled momentum equation and regular NS, but need not necessarily dramatically alter the bound. The impact on the bound depends on the effect of the pressure term (through the pressure-dilatation) on the energy dissipation in supersonic turbulence. As an example, consider the isothermal scenario ($\gamma = 1$). In this case, the scaled momentum equation becomes

$$\frac{\partial \hat{\mathbf{v}}}{\partial \hat{t}} = \hat{\mathbf{v}} \cdot \nabla \hat{\mathbf{v}} - \bar{L}^2 \frac{K}{\hat{\rho}} \nabla \hat{\rho} + \bar{L}^3 \hat{\mathbf{D}}. \quad (19)$$

The decay rate of compressible turbulence is the result of the net effect of the viscous dissipation, $\hat{\mathbf{D}}$, and the pressure-dilatation, which comes from the dot product of the pressure-gradient term with $\hat{\mathbf{v}}$. To the extent that the pressure-dilatation enhances the decay rate, the bound should be insensitive to the \bar{L}^2 scaling. This is because the bound comes about by considering $\bar{L} = 1$ to be a more dissipative case than when \bar{L} shrinks in time, which would remain the case. There is some evidence the pressure-dilatation does in fact increase the dissipation in decaying turbulence, at least in the subsonic

case (Sarkar 1992; Samtaney et al. 2001). Even without this, the bound will be approximately preserved so long as the net effect on the decay of the pressure-dilatation term with the \bar{L}^2 coefficient is small compared to that of the viscous dissipation term with the \bar{L}^3 coefficient. For the isothermal case the pressure term scales as the sound speed squared, C_s^2 , which becomes small in the high-Mach limit. However, for very large compressions (reaching very small \bar{L}), the weaker decrease on the pressure term may relatively enhance its contribution even if it would normally be small.

Overall, even for $\gamma = 5/3$, the bound can only be universal to the extent that the decay of supersonic turbulence is (Federrath 2013). To the extent the mix of compressible and solenoidal modes in the initial condition affects the decay rate, this must be accounted for in the value of α . Similarly with the impact of changing γ and changing initial mach number.

As noted in Sec. 2, the present treatment considers contractions where the time dependence of \bar{L} is determined by the gravitational attraction of the mean density. Strictly speaking, if \bar{L} is taken to have a different form, one must consider the effect of an acceleration term $\ddot{\bar{L}}\mathbf{x}$ in the momentum equation, Eq. (5). This may or may not have a significant impact on the bound. As also noted in Sec. 2, gravitational effects from the density fluctuations have been neglected. In many astrophysical problems of interest, there is forcing besides for the contraction which acts on the turbulence, which is neglected here.

These various assumptions and restrictions, if limiting to the generality of the bound, should be replicable for simulations. Then the bound provides a relatively simple, high level check on the simulations, particularly on the degree of dissipation. The initial application of the

bound in this manner suggests a commonly used model (and matching simulations) may be too dissipative. Note that, even if a simulation or model is too dissipative, it may still be useful, depending on the physics under consideration.

The implications of the rescaled equations, Eqs. (14,15), apart from the bound, deserve mention. These equations are reached because forcing of the type generated by contraction can be scaled out of the NS equations. The only difference between the rescaled equations and compressible NS equations is in the dissipation term. However, many turbulent quantities, for example, inertial range properties, are not influenced by the dissipation properties of the turbulence. Therefore, we may expect that already known results for decaying supersonic turbulence can be translated by undoing the scaling, and applied to turbulence undergoing compression. This task is made simpler by the fact that the rescaling, Eqs. (9,10,11) is purely time dependent, so that, for example, spatial correlation functions are translatable.

In conclusion, we have suggested a lower bound on the increase in turbulent velocity associated with compression of turbulence. This lower bound follows directly once one assumes that a decreasing-in-time coefficient of viscosity in the NS equations does not increase the rate of dissipation for turbulence. This assumption, while physically reasonable, should be verified, or disproved, since the bound represents a useful means of checking models or simulations of compressing turbulence, and an initial application of the bound in this capacity indicates some previous work may be too dissipative.

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